(i) **Printed Pages: 2** Roll No. ....

(ii) **Questions : 8** **Sub. Code : 0 4 4 4**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

(1129)

MATHEMATICS

Paper : II (Modern Algebra)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Question paper will consist of **EIGHT** questions. Candidate will attempt **FIVE** questions in all, selecting at least **TWO** questions from each Unit. All questions carry equal marks.

UNIT—I

1. (a) Show that the set of all positive rational numbers under the composition defined by $a * b = \frac{ab}{3}$ forms an infinite abelian group.

   (b) If G is a finite group of order n then prove that for any $a \in G$, $\exists$ some positive integer r, $1 \le r \le n$, such that $a^r = e$. 3,3

2. (a) Let G be a group and let $a \in G$ be order m. Then prove that $O(a^k) = \frac{m}{(m, k)}$, where $k \in N$.

(b) Prove that if G be a finite group and $a \in G$. Then $O(a)/O(G)$ i.e., the order of an element of a group is a divisor of the order of the group. 3,3

3. State and prove Cayley's Theorem. 6

4. (a) Prove that a group of order n is cyclic iff it has an element of order n.

(b) Prove that group of prime order must have a non-trivial centre. 3,3

UNIT—II

5. (a) Show that the set of rational numbers Q is a ring under the compositions $\oplus$ and $\odot$ defined as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$, $\forall$ $a, b \in Q$.

(b) Let R be a ring such that $x^3 = x$ $\forall$ $x \in R$. Prove that R is a commutative ring. 3,3

6. (a) Show that if $1 - ab$ is invertible in a ring R with unity 1 then so is $1 - ba$ and that $(1 - ba)^{-1} = 1 + b(1 - ab)^{-1}a$.

(b) Let $I = (a)$, $J = (b)$ be two ideals of the ring Z of integers, where a, b are positive integers. Determine (i) $I + J$, (ii) $I \cap J$, (iii) IJ. 3,3

7. An ideal M of a commutative ring with unity is a maximal ideal iff R/M is a field. 6

8. (a) What are the units of the polynomial ring $Z_7[x]$?

(b) Show that ideal $< x >$ of Z[x] is prime ideal. 3,3

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